Math 136.101 Midterm B

Instructions:

1. Answer all questions on the machine-scored answer sheet provided. Use pencil only.
2. Return examination paper with machine-scored answer sheet.
3. Single-line display calculators permitted. No other aids permitted.
4. Fill in the information requested below.
5. The examination invigilators may not interpret or explain questions to you.
6. Fill in your student number on the machine-scored sheet and encode it as well.

FAMILY NAME _____ **FIRST NAME** _____

STUDENT NUMBER _____

SIGNATURE _____

INSTRUCTOR _____

1. The accompanying matrix is not in row echelon form.
What is the reason?

(a) too many leading 1's
(b) not enough 0's above leading 1's
(c) Not enough 0's below leading 1's
(d) not all leading 1's in correct places
(e) the matrix is actually in row echelon form

$$\begin{bmatrix} 1 & 0 & 3 & 2 & 3 & 6 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & -4 & 7 \\ 0 & 0 & 0 & 1 & -3 & -2 \end{bmatrix}$$

2. What **single** elementary row operation would simplify the accompanying matrix to row reduced echelon form?

(a) interchange two rows
(b) interchange two columns
(c) add a multiple of one row to another row
(d) multiply a row by a nonzero constant
(e) none of the above

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 4 \\ 0 & 1 & 3 & -2 & 1 \\ 0 & 0 & 1 & 2 & 4 \end{bmatrix}$$

3. When a row echelon form for a matrix is different than the row reduced echelon form for the same matrix, the difference is:

(a) the number of leading 1's
(b) the number of rows of 0's
(c) the positions of leading 1's
(d) the number of 0's in columns with leading 1's
(e) none of the above

4. You are given that the row reduced echelon form for the augmented matrix associated with the system of equations

$$\begin{aligned} x - 2y + 3z &= 4 \\ 2x + y + z &= -2 \\ 3x - y + 4z &= 2 \end{aligned}$$

is

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

The solution of the system is:

(a) $x = 0$, $y = -2$, $z = 0$
(b) $x = -s$, $y = s - 2$, $z = s$, where s is arbitrary
(c) $x = s$, $y = -s - 2$, $z = s$, where s is arbitrary
(d) the system does not have a solution
(e) none of the above

5. Which one of the following statements is correct?

(a) All matrices have inverses.
(b) All square matrices have inverses.
(c) Some non-square matrices have inverses.
(d) Some square matrices have inverses.
(e) None of the above statements is correct.

6. Which of the following matrices is the row reduced echelon form for the augmented matrix to the right?

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & -3 & 5 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 6 \end{array}\right)$$

(a) $\left(\begin{array}{cccc|c} 1 & 0 & 2 & -3 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 1 & 2 & 1 \end{array}\right)$

(b) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & -7 & -1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$

(c) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 11 \\ 0 & 1 & 0 & 4 & 4 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$

(d) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 3 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right)$

(e) None of the above

7. When the augmented matrix for a system of three equations in 4 unknowns is simplified, the row echelon form is shown to the right. What do you conclude about the number of solutions of the equations?

$$\left(\begin{array}{cccc|c} 1 & 6 & 2 & -3 & 5 \\ 0 & 1 & 3 & 4 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

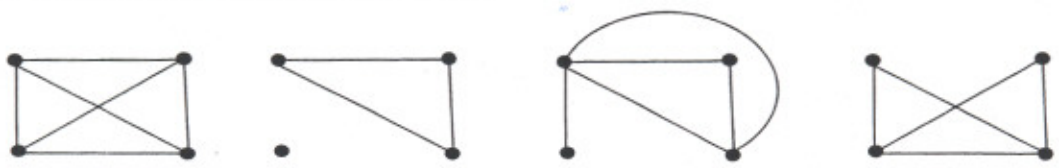
(a) There is exactly one solution.
(b) There is no solution.
(c) There is an infinity of solutions.
(d) There is not yet enough information to conclude how many solutions the system has.
(e) None of the above

8. If the augmented matrix to the right is simplified to row (reduced) echelon form, how many leading 1's does it have?

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 3 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 2 & 8 \end{array}\right)$$

(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

9. Which of the following graphs is not simple?



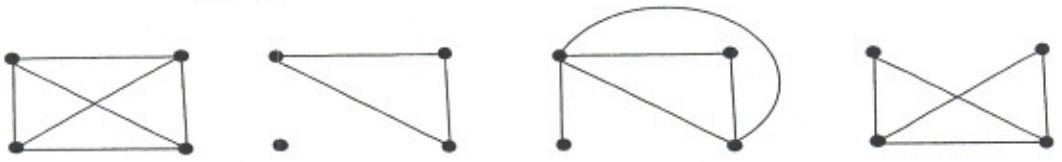
(a) (b) (c) (d)

(e) They are all simple.

10. A graph is known to have 19 edges and 5 nodes with degree 4. If all other nodes have degree 3, how many nodes does the graph have in total?

(a) 11 (b) 6 (c) 17 (d) 38 (e) None of these

11. Which of the following graphs has an isolated node?



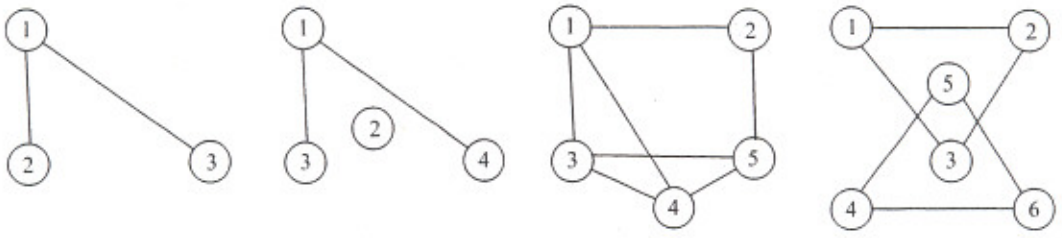
(a) (b) (c) (d)

(e) None of them has an isolated node.

12. If the degrees of the nodes of a graph are $\{2,3,1,2,4\}$, how many edges does it have?
(a) 6 (b) 24 (c) 12 (d) 17 (e) We cannot tell.

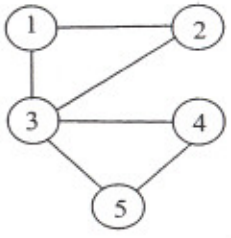
13. Which of the following is not possible as the degrees for the nodes of a simple graph?
(a) $\{2,1,3,2\}$ (b) $\{0,1,3,2,2\}$ (c) $\{2,2,2,2\}$ (d) $\{1,2,2,2\}$
(e) All of these are possible

14. How many of the following graphs are connected?



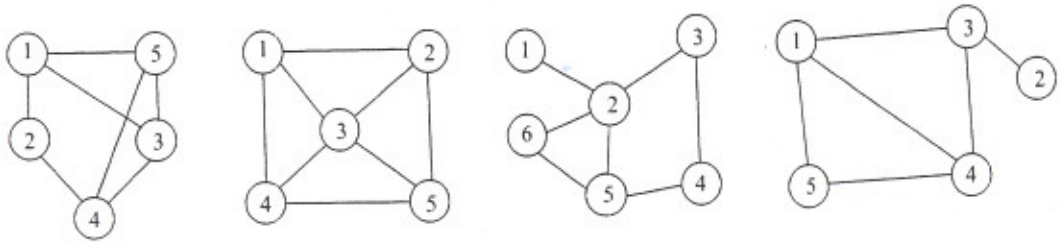
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

15. Which of the following is not an Euler circuit for the graph to the right?



(a) 3,2,1,3,5,4,3 (b) 2,1,3,5,4,3,2
(c) 1,2,3,4,5,3,1 (d) 1,3,2,1,3,2,1
(e) All of the above are Euler circuits.

16. Which of the following graphs has an Euler path?



(a) (b) (c) (d)

(e) None of the above

17. In which of the following situations does a connected graph, with no isolated nodes, have an Euler path but does not have an Euler circuit?

(a) All nodes have even degree.

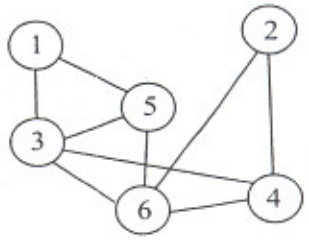
(b) All nodes have even degrees except for an even number of nodes that have odd degrees.

(c) All nodes have even degrees except that exactly two nodes have odd degrees.

(d) All nodes have even degrees except that exactly one node has odd degree.

(e) None of the above

18. Which of the following is a Hamiltonian circuit for the graph to the right?

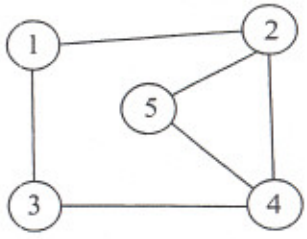


(a) 1,5,2,4,6,3,1 (b) 2,4,6,3,1,5,6,2

(c) 5,6,4,2,6,3,1,5 (d) 1,5,6,2,4,3,1

(e) None of the above

19. Which of the following is the adjacency matrix for the graph to the right?

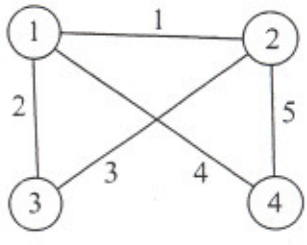


(a) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

(e) None of the above

20. Which of the following is the incidence matrix for the graph to the right?



(a) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

(e) None of the above

21. If V is the adjacency matrix associated with a simple graph, then the $(i,j)^{th}$ entry in V^3 is:

(a) the number of edges joining nodes i and j

(b) the number of directed edges from node i to node j

(c) the number of routes of exactly 3 edges from node i to node j

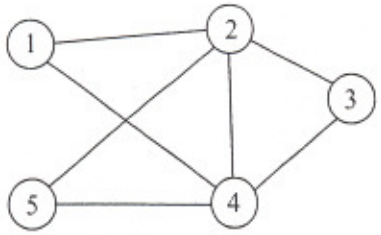
(d) the number of routes of not more than 3 edges from node i to node j

(e) none of the above

22. Which of the following is a correct statement?

(a) The adjacency matrix of a graph must contain only 0's and 1's.
(b) The adjacency matrix of a digraph contains negative as well as postive integers.
(c) The adjacency matrix of a graph must have a 0 in the $(1, 1)$ position.
(d) The adjacency matrix of a graph must be square.
(e) None of the above

The last three questions refer to the graph below.



You are given the adjacency matrix V for the above graph and V^2 and V^3:

$$V = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \quad V^2 = \begin{pmatrix} 2 & 1 & 2 & 1 & 2 \\ 1 & 4 & 1 & 3 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 4 & 1 \\ 2 & 1 & 2 & 1 & 2 \end{pmatrix} \quad V^3 = \begin{pmatrix} 2 & 7 & 2 & 7 & 2 \\ 7 & 6 & 7 & 7 & 7 \\ 2 & 7 & 2 & 7 & 2 \\ 7 & 7 & 7 & 6 & 7 \\ 2 & 7 & 2 & 7 & 2 \end{pmatrix}$$

23. Which of the following statements is **not** correct?

(a) There are exactly 4 routes of at most 2 edges from node 3 to node 5.
(b) There are exactly 2 routes of 2 edges from node 3 to node 5.
(c) There are exactly 2 routes of 3 edges from node 3 to node 5.
(d) There are exactly 2 routes of 3 edges from node 1 to node 5.
(e) All of the above statements are correct.

24. The number of routes of at most 2 edges from node 2 to node 3 is:

(a) 1 (b) 2 (c) 3 (d) 4 (d) None of these

25. The 3 in position $(2, 4)$ of V^2 identifies the following routes in the graph.

(a) $2 \to 4 \to 2 \to 4$, $2 \to 3 \to 2 \to 4$, $2 \to 1 \to 2 \to 4$, $2 \to 5 \to 2 \to 4$
(b) $2 \to 4 \to 5 \to 4$, $2 \to 4 \to 3 \to 4$, $2 \to 4 \to 1 \to 4$
(c) $4 \to 3 \to 2$, $4 \to 1 \to 2$, $4 \to 5 \to 2$
(d) $2 \to 3 \to 4$, $2 \to 1 \to 4$, $2 \to 5 \to 4$
(e) None of the above